Name: _____

1. (20 marks) Study the convergence of $\sum_{k=2}^{\infty} \dfrac{\cos k^2}{k^2 - 1}$.

   Solution. We have, for $k \geq 2$,

   $$
   \begin{aligned}
   |x_k| &\equiv \left| \frac{\cos k^2}{k^2 - 1} \right| \\
   &\leq \frac{1}{k^2 - 1} \\
   &\leq \frac{2}{k^2}.
   \end{aligned}
   $$

   As $\sum 2/k^2$ is convergent, by Comparison Test $\sum_{n=1}^{\infty} x_n$ is absolutely convergent.

   Remark. This shows the power of the comparison test. Some of you tried Ratio Test, but it is no good. On the other hand, you may apply Integral Test.